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Modeling of capillary flow in shaped polymer fiber bundles

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Abstract Moisture transporting in fiber assembly is one of the critical factors affecting physiological comfort. In this study, we investigated at the capillary flow in complex geometries representative of the void spaces formed between fibers in shaped polymer fiber bundles. Dynamic process of liquid creeping in capillary is analyzed based on Reed and Wilson vertical wicking model. Critical equivalent radius values of capillary tubes in polymer fiber assembly are discussed here. In the cases of round, criss-cross and triangle shape fiber, Reed and Wilson model is integrated with shaped fiber bundle mathematical simulation model (MFB) to calculate the dynamic curve of liquid arising. Instantaneous wicking velocity, wicking height, wicking flux, and characters are compared to figure out that the wicking effect of shaped polymer fiber is quite better than normal round one.

Introduction

Liquid transporting in fiber assembly is one of the most important factors affecting the physiological comfort of clothing fabric [1, 2]. So it is significant to improve the wicking ability of hydrophobic fabric for the purpose of high wicking and quick drying. It is useful for understanding the mechanism of liquid flowing in fiber assembly to investigate the geometric distribution of capillary inter-space in polymer fiber assembly, which affect the results of liquid quantity, wicking time of capillary action. This study focus on the liquid flow in shaped fiber bundle explained by the surface tension driven mathematical model.

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Capillary action depends on the liquid property and surface specialty of fiber material, like liquid viscosity, surface tension as well as the geometric shape of capillary space [3, 4]. For the hydrophobic polymer fiber, the main approach for liquid transporting in fiber assembly is inter space between fibers in yarn rather than the interstice between yarns in fabric.

Because of the complexity of microscopic structure of fabric and the transferability of fibers in yarn or fabric during processing and application, it is difficult to predict the structure of capillary space as well as its distribution. For the nature fiber material, wetting will even make the fiber body swelled, transformed or transferred, so as to change the capillary space position. So it will be easier to understand how liquid flow in single capillary tube before the complicated alignment of capillaries in fabric and the liquid absorbing capability are concerned.

Some experts have done relative work about liquid wicking in round conduit like Washburn, Reed and Wilson [5, 6]. They get conclusion that the way of liquid flow into capillary tube and the curved surface of liquid in the capillary doesn't affect the capillary action very much. Reed and Wilson deduced the physical equation of liquid arising along the single vertical capillary tube to explain the dynamic mechanics of liquid transporting.

When the theory of single capillary with round cross section is applied into the capillaries in fiber assembly, there will be some problems. Even the cross sectional shape of single fiber is round one, the shape of capillary space forming between fibers is much different with the regular round. If the cross sectional shape of fiber changes, the capillary shape will transform more complicatedly. In this work, we simulated the stochastic alignment of filament bundles with multi-shapes by building the mathematical models of bundle's cross section. Analyzing or computation of capillary equivalent radius, liquid height, flux, wicking velocity, and time will describe the dynamic arising procedure numerically.

Analyzing and calculating model

Vertical wicking model for polymer fiber bundle

Reed and Wilson introduced vertical capillary model which is shown is Fig. 1 [5].

According to R&W Model, Fig. 2 shows the dynamic forces balance.

$$\sum P = p_A + p_T + p_G + p_v + p_I \quad (1)$$

That is:

$$p_A + CT\cos\theta - A\rho gx - k(x + x_0)\partial x/\partial t - \partial/\partial t[A\rho(x + x_0)\partial x/\partial t] = 0 \quad (2)$$

where k is equal to 8 $\pi\eta$ from Hagen–Poiseuille law (Table 1).

When the wicking action is considered for fiber assembly, people used to take "open array" as the model of fiber bundle (MFB) [3]. However, in the cross section of

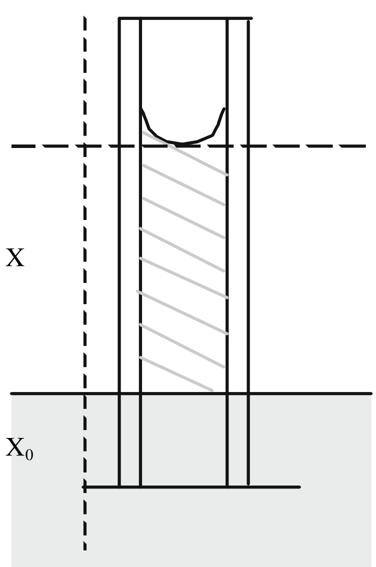

Fig. 1 R&W Model

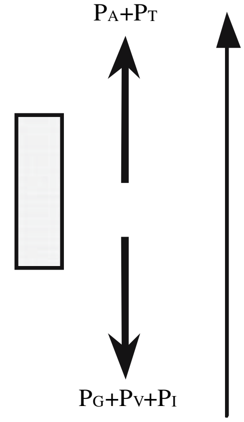

Fig. 2 Kinetic analysis model of vertical capillary

yarn, fibers are shown that they are arrayed randomly, especially for bundles with exactly the same individual fibers. Here, we put forward a new MFB shown in Fig. 3 [7].

Model of Fiber Bundle is assumed as one micro-unit of yarn or filament tow. In MFB, fibers are paralleled with each other and the alignment of fibers in bundle is stochastic. The single fiber is rigid so the fiber will never transform because of pressure. And the cross sectional shape of single fiber can be round, triangle, criss-cross and some other special designed shape shown in Fig. 4. In the calculation of capillary flow in fiber bundle, we will compare the regular round one and shaped one to figure out which kind of shaped fiber has better wicking ability.

Maximum height of arising capillary

In MFB, all capillaries forming between fibers are open, so P_A is neglected here. Thus the equation of liquid flowing in round fiber bundle will be:

Table 1 Code parameter

P_A: Atmospheric pressure	T: Surface tension
P_T: Surface tension force	η: Liquid viscosity
P_G: Liquid gravity	θ: Contact angle
P_v: Liquid viscidity force	A: Cross section area of capillary
P_I: Liquid inertia force	C: Perimeter of capillary cross section
ρ: Liquid density	X_0: Iinitialization height of capillary liquid
X: Height of arising capillary	$\hat{R}$: Equivalent radius of capillary
R: Radius of fiber bundle	r: Radius of single fiber
N: Fiber number	n: Capillary number
(R): Curved surface radius	H: Critical height of capillary
t: Wicking time	

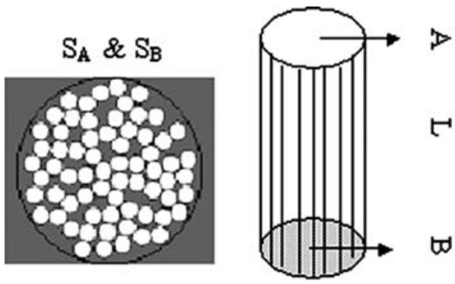

Fig. 3 Vertical wicking MFB

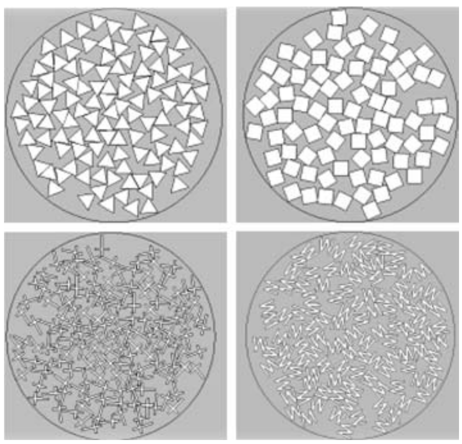

Fig. 4 Cross sections of shaped fiber bundle

$$\left(\sum_{i=1}^{N} 2\pi r_i\right) T\cos\theta - \left(\pi R^2 - \sum_{i=1}^{N} \pi r_i^2\right)\rho g x - k(x+x_0)\partial x/\partial t - \partial/\partial t[A\rho(x+x_0)\partial x/\partial t] = 0 \quad (3)$$

When the time is long enough, the fiber bundle can be saturated wicked. So the capillary liquid can reach a steady state height:

$$x_\infty = \frac{\sum_{i=1}^{N} 2\pi r_i T\cos\theta}{(\pi R^2 - \sum_{i=1}^{N}\pi r_i^2)\rho g} \quad (4)$$

Equation 4 demonstrates that the geometric size and morphology affect the result of capillary consequently when R and other parameters are given.

Wicking time

On the case of assuming the acceleration is zero (inertia is negligible), Eq. 2 can be written in:

$$\mathrm{C}T\cos\theta - A\rho g x - k(x+x_0)\partial x/\partial t = 0 \quad (5)$$

where $\partial/\partial t[A\rho(x+x_0)\partial x/\partial t] = 0$ and capillaries are open.

The explicit form of Eq. 5 will be: (Initial condition: $t = 0$, $x = 0$)

$$t = \frac{8\eta}{\rho g \hat{R}^2}\left[(x_\infty + x_0)\ln\left(\frac{x_\infty}{x_\infty - x}\right) - x\right] \quad (6)$$

Equation 6 can be written in more concise:

$$t = \frac{8\eta}{\rho g \hat{R}^2}\left[(x_\infty + x_0)\ln\left(\frac{x_\infty}{x_\infty - x}\right) - x\right] = \frac{1}{\hat{R}^2}\mathrm{MF}(x) \quad (7)$$

Therefore, the wicking time for fiber bundle can be expressed as:

$$t_\mathrm{P}: \left[\frac{1}{\hat{R}_{\max}{}^2}\mathrm{MF}(x), \frac{1}{\hat{R}_{\min}{}^2}\mathrm{MF}(x)\right] \quad (8)$$

Here, $\hat{R}_{\max}$ and $\hat{R}_{\min}$ present the radius of maximum and minimum capillary in fiber bundle separately. Hence, the wicking time for a fiber bundle will be in the period of t_P theoretically.

Instant wicking velocity

From Eq. 7, the maximum wicking velocity of single capillary when $t = 0$ will be found in:

$$\left.\frac{\partial x}{\partial t}\right|_{t=0} = \frac{\rho g \hat{R}^2 x_\infty}{8\eta x_0} \quad (9)$$

For a fiber bundle Eq. 9 can be:

$$\overline{V}_0 = \left.\overline{\frac{\partial x}{\partial t}}\right|_{t=0} = \frac{\rho g x_\infty}{8\eta x_0}\sum_{i=1}^{n}\hat{R}_i{}^2 \quad (10)$$

Since the instant moisture absorbing ability when the fabric contact with skin affects the physiological comfort significantly, $\overline{V}_0$ can be taken as an important evaluating parameter for the characterization of fabric made of shaped fibers.

Reynold's experience criterion of flow in capillary has the condition that [8]:

$$\frac{2\hat{R}\rho}{\eta}\cdot\frac{\partial x}{\partial t} \le 2{,}000 \quad (11)$$

Equation 11 illustrates that the arising velocity is proportion inversely to the equivalent radius of capillary. While Eq. 9 demonstrates that the instant wicking velocity is proportion to the square of equivalent radius.

When the big capillaries are much more than the small ones, the bundle will have good ability of instant moisture absorption but slower arising velocity. And the maximum height of liquid arising will also go downwards. If the value of capillary equivalent radius distributes in appropriate way, the instant wicking velocity and the arising velocity will be in optimization synchronously.

Critical value of capillary equivalent radius

Normally the inter-place formed between yarns has the function of air permeability, not the liquid flowing driven by capillary pressure. Some capillary pores formed between fibers cannot work because of their limited micro-size. In this section, the efficiency geometric size of inter-place formed between fibers is discussed elementarily.

According to the capillary equivalent radius and the wicking length value issued by Washburn, the minimum capillary equivalent radius $\hat{R}_{\min}$ can be predicted well founded.

Data in Table 2 shows that the wicking length is less than the equivalent radius by three orders of magnitude on the condition of $\hat{R} < 0.01$ μm. So the wicking length can be neglected when $\hat{R} < 0.01$ μm, namely $\hat{R}_{\min}$ can be set as 0.01 μm.

The capillary space is assumed to be cylinder. The critical status for liquid arising in the maximum cylindrical capillary (meniscus) is shown as Fig. 5. If $H \ll (R)$, the liquid just has a trend to arise along the mural surface of capillary not to form a capillary. When (R) = H, the capillary is in the critical height. So $\hat{R}_{\max}$ is set as $\hat{R}_{\max} = (R) = H$. If $r > \hat{R}_{\max}$, there will be no cylindrical capillary.

The force balance in Fig. 5 is:

Table 2 capillary equivalent radius and wicking length

$\hat{R}$ (mm)	1	0.1	0.01	0.001
l_0 (mm)	2	0.04	0.0004	0.000004

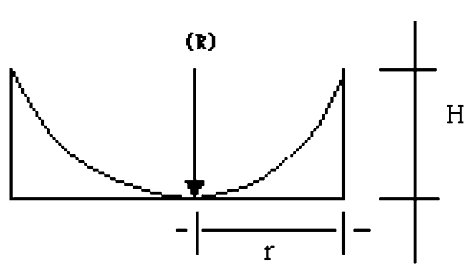

Fig. 5 Critical capillary tube

$$\mathrm{CT}\cos\theta = \pi r^2 H\rho g - \frac{2}{3}\pi r^3 \rho g + P_{\mathrm{A}} \tag{12}$$

When $(R) = H = \hat{R}_{\max}$, Equation 12 can be written as:

$$\frac{1}{3}\pi \hat{R}_{\max}{}^{3}\rho g - 2\pi \hat{R}_{\max} T\cos\theta + P_{\mathrm{A}} = 0 \tag{13}$$

$\hat{R}_{\max}$ is given by liquid and solid surface material. Here $\theta = 60^{\circ}$, $T = 7.2 \times 10^{-3}$ N/m, $\rho = 1{,}000$ Kg/m, $g = 9.82$ m/s^2, $\eta = 1 \times 10^{-3}$ Pa s, $P_{\mathrm{A}} = 1.01 \times 10^5$ Pa. The result of $\hat{R}_{\max}$ is shown to be much bigger than interstice formed in fabric.

Result and discussion

By analyzing the images simulated from MFB, the parameters of capillaries in fiber bundle are acquired in Table 3.

Table 3 demonstrates that the maximum height of arising capillary for crossed shape fiber bundle is the highest one. The second one is double-crossed. The square one as well as round one is shown to be much lower. So the cross sectional shape with grooves of single fiber has better liquid wicking ability.

Taking the arising height as 0.15 m, Eqs. 6 and 8 can give the minimum liquid arising time for the five fiber bundles. The wicking time of double-crossed and crossed shaped fiber bundles are 288.8 and 308.3 s respectively, which are much less than the other three bundles (1,000–2,000 s).

The dynamic relationship between wicking time and liquid arising height for all the capillaries in bundle can be expressed as:

$$t = \frac{8n\eta}{\rho g \sum_{i=1}^{n} \hat{R}_i^2}\left[(x_\infty + x_0)\ln\left(\frac{x_\infty}{x_\infty - x}\right) - x\right] \tag{14}$$

The dynamic liquid flowing equations of five typical shaped fiber bundles are listed in Table 4. Figure 6 illustrates the dynamic arising curves. It can be found that the arising heights of round, triangle and square shaped fiber bundle are comparably lower and they tend to be stopped at the value of 0.2–0.25 m. While the crossed and double-crossed shaped fiber bundles have much better trend of arising higher.

Figure 7 shows the dynamic flow in the time of 50 s. By comparing the slope, the consequence of instant wicking velocity is found to be:

Cross > Double-cross > Triangle > Square > Round

Table 3 Parameters of capillary inter-place

Fiber type	Round	Triangle	Square	Cross	Double cross
N	68	118	87	152	140
n	30	36	23	57	90
$\hat{R}_{\max}(10^{-6}\text{m})$	6.6747	6.0434	5.3475	7.6864	8.3412
$\hat{R}_{\min}(10^{-7}\text{m})$	6.9868	8.2514	3.4002	3.4002	3.7591
x_∞	0.3116	0.2795	0.2575	0.6130	0.5655
$\sum_{i=1}^{n} \hat{R}_i(10^{-5}\text{m})$	6.8404	10.2507	6.3891	12.6371	14.9523
$\sum_{i=1}^{n} \hat{R}_i^{\,2}(10^{-10}\text{m}^2)$	2.3099	3.6969	2.2507	4.4822	5.0170
t_P (s) (x = 0.15 m)	[1012.4, ∞]	[1470.6, ∞]	[2164.1, ∞]	[308.3, ∞]	[288.8, ∞]
$\overline{V}_0(10^{-3}$ m/s)	2.939	3.516	3.087	5.905	3.862

Table 4 Dynamic equations of fiber bundles wicking

Fiber type	Dynamic equation of fiber bundle wicking
Round	$t1 = 4987.6^*(0.3126^*\ln(0.3116/(0.3116–x))–x)$
Triangle	$t2 = 2701.75^*(0.2805^*\ln(0.2795/(0.2795–x))–x)$
Square	$t3 = 4359.12^*(0.2585^*\ln(0.2575/(0.2575–x))–x)$
Cross	$t4 = 2834.1546^*(0.614^*\ln(0.613/(0.613–x))–x)$
Double-cross	$t5 = 3213.656^*(0.5665^*\ln(0.5655/(0.5655–x))–x)$

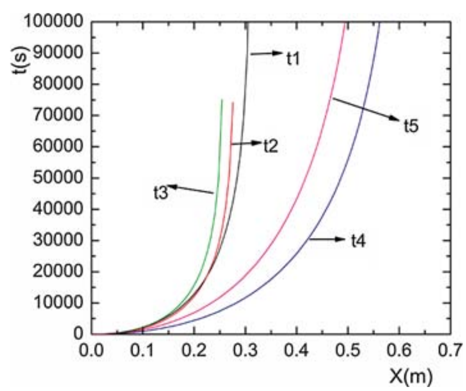

Fig. 6 Dynamic curves of fiber bundles wicking

The initial wicking velocity $\overline{V}_0$ in Table 3 is also shown to be the quickest one (5.905 × 10^{-3}m/s). And the round one is only 2.939 × 10^{-3}m/s.

Conclusions

The combination application of kinetic analysis model of vertical capillary and MFB for fiber bundle is an efficiency method for simulating liquid flow in fiber assembly. MFB can simulate the cross section of shaped fiber bundle and predict the affection of capillary space formed between fibers on the wicking action of fiber bundle. By analyzing and calculating the parameters of fiber bundles like fiber number, wicking height, instant wicking velocity as well as wicking time, the shaped fiber bundles like cross or double-cross shaped one are considered as the optimized shaped fiber bundle which is much advantageous for liquid transporting.

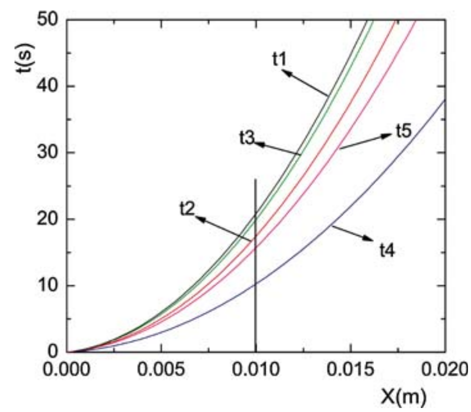

Fig. 7 Initial instantaneous wicking velocity of fiber bundles

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